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An EOQ Model of Deteriorating Products with Inventory Level Dependent Demand Rate under Trade Credit and Time Discounting

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ABSTRACT:
In this paper, an EOQ model has been developed for deteriorating products with life time, inventory level dependent demand rate and time dependent demand rate under trade credit. Three components demand rate has been considered. Deterioration rate has been taken constant. Permissible delay in payments is allowed and inflation has been considered.

Key-words: EOQ model, deterioration, time discounting and trade credit.

INTRODUCTION:
In traditional inventory models the demand rate is assumed to be constant. It is observed that the stock level may influence the demand rate in the case of some consumer products. It is common experience that displaced stock level attracts more consumers. This observation inspired the researchers to consider the stock dependent demand rate. Several researchers such as **Level et al.** (1972), **Silver** (1981) and **Silver & Peterson** (1985) worked in this direction.
Gupta & Vrat (1986) discussed the stock dependent demand rate inventory model. Calculation of average system cost was not correct in this paper. **Mandal & Phaujdar** (1989) suggested the correction to the average system cost. **Bakar and Urban** (1988) provided the first rigorous attempt in developing the stock dependent demand rate. The functional form presented by them is realistic and logical from practical as well as economic point of view.

Dutta & Pal (1990) developed an inventory model with stock dependent demand rate using some functional form as taken by **Bakar and Urban** (1988). **Dutta & Pal** (1990) developed another model for deteriorating items with demand rate dependent on inventory level with shortages. **Mandal & Phaujdar** (1989) also developed another model for deteriorating items with stock dependent demand rate, variable rate of deterioration and shortages fully backlogged. **Sarkar, et al.** (1997) introduced the realistic concept of decreasing demand. **Jain and Kumar** (2007) developed an inventory model with stock level dependent demand rate, shortages and decrease in demand. They considered two level demand rate for infinite time horizon as well as finite time horizon. **Hou and Lin** (2008) developed an ordering policy for deteriorating items under trade credit and time discounting. **Hari Kishan, Megha Rani and Deep Shikha** (2012) discussed the inventory model of deteriorating products with life time under declining demand and permissible delay in payment.

This paper deals with an EOQ model for deteriorating products with life time, inventory level dependent demand rate and time dependent demand rate under trade credit. Three components demand rate has been considered. Deterioration rate has been taken constant. Permissible delay in payments is allowed and inflation has been considered.

Notations and Assumptions

Notations:

The following notations have been used in this paper:

$q(t)$: Inventory level at any time t.

S: Stock level at the beginning of each cycle after fulfilling the backorders.

c_1: The holding cost per unit per unit time.

c_2: The shortage cost per unit per unit time.

A: Ordering cost per order.

μ: The time up to which there is no deterioration

T: The length of each cycle time.

T_0: Time period in which there is inventory in the system.

M: Time period for permissible delay in payment.

R: The net discount rate of inflation.

C_H: The total holding cost of inventory in the interval $[0, T_0]$.

C_S: The total shortage cost of inventory in the interval $[T_0, T]$.

C_D: The total holding cost of inventory in the interval $[\mu, T_0]$.

c: Unit purchasing cost.

s: Unit selling price.

I_c: Interest charged per Re per unit time.

I_e: Interest earned per Re per unit time.

Assumptions:

The following assumptions have been considered in this paper:

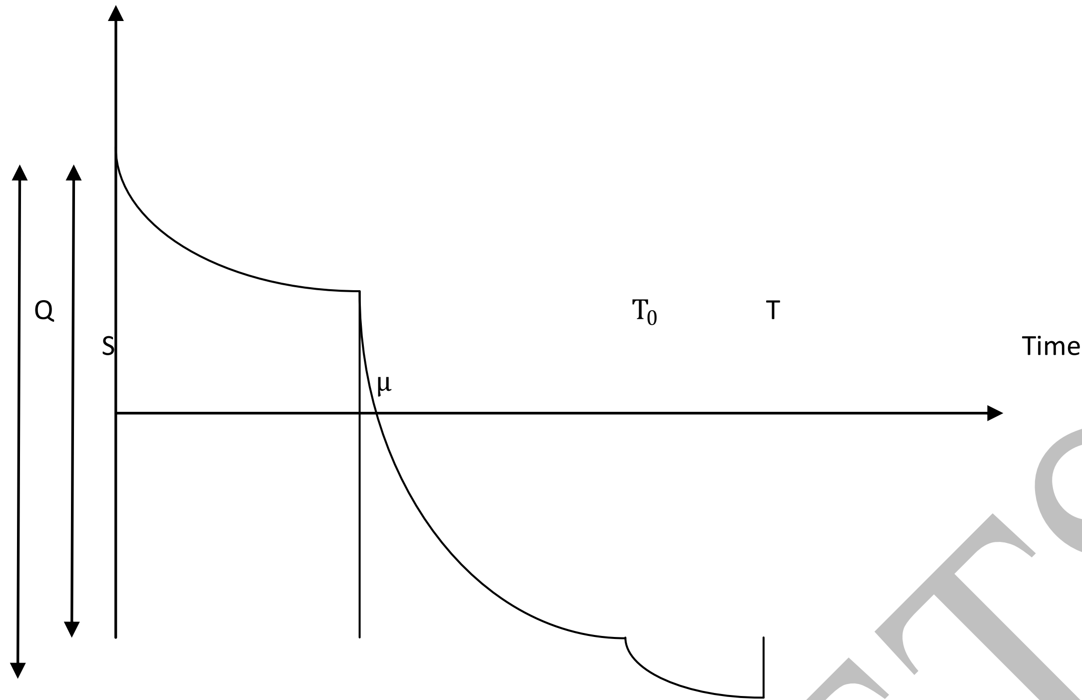
1. There is single item in the inventory system.
2. Lead time is negligible. It is considered as zero.
3. Replenishments are instantaneous, i.e. the replenishment rate is taken infinite.
4. Shortages are allowed and fully backlogged.
5. These components demand rate is deterministic and known function of instantaneous inventory level upto a certain interval of time and after that the demand rate is time dependent. Thus the demand rate is given by

$$\frac{dq}{dt} = \begin{cases} (\alpha + \beta q), & 0 \le t \le \mu \\ (\alpha + \gamma q), & \mu \le t \le T_0 \\ (\alpha + \delta(t - T_0)), & T_0 \le t \le T \end{cases}$$

Mathematical Model and Analysis:

Let Q be the number of items received at the beginning of the cycle. $(Q - S)$ items are delivered for the fulfillment of backorder leaving a balance of S as the initial inventory of new cycle. The inventory level depleted at a rate of $(\alpha + \beta q)$ during the period$[0, \mu]$. During the period $[\mu, T_0]$ the inventory level depleted at the rate$(\alpha + \gamma q)$. The inventory level falls to zero at time T_0. Shortages are allowed for replenishment upto time T. The model has been shown in the (Figure 1) given below:

Inventory



(Figure 1)

The inventory system is governed by the following differential equations:

$$\frac{dq}{dt} = -(\alpha + \beta q), \qquad 0 \le t \le \mu \qquad \dots(1)$$

$$\frac{dq}{dt} + \theta q = -(\alpha + \gamma q), \qquad \mu \le t \le T_0 \qquad \dots(2)$$

$$\frac{dq}{dt} = -(\alpha + \delta(t - T_0)), \qquad T_0 \le t \le T \qquad \dots(3)$$

where $\alpha, \delta > 0$ and $0 < \gamma < \beta < 1$.

With the following boundary conditions:

$$q(0) = S,\ q(T_0) = 0 \text{ and } q(T) = -(Q - S). \qquad \dots(4)$$

Solution of equations (1), (2) and (3) with the help of boundary conditions (4) are respectively given by

$$q = \left(S + \frac{\alpha}{\beta}\right) e^{-\beta t} - \frac{\alpha}{\beta}, \qquad 0 \le t \le \mu \qquad \dots(5)$$

$$q = \frac{\alpha}{\theta+\gamma}\left[e^{(\theta+\gamma)(T_0 - t)} - 1\right], \qquad \mu \le t \le T_0 \qquad \dots(6)$$

$$q = -\alpha(t - T_0) + \frac{\delta}{2}(t^2 - T_0^2). \qquad T_0 \le t \le T \qquad \dots(7)$$

From expressions (5) and (6), we get

$$S = \frac{\alpha}{\theta+\gamma}\left[e^{(\theta+\gamma)(T_0-\mu)} - 1\right]e^{\beta\mu} + \frac{\alpha}{\beta}(e^{\beta\mu} - 1). \qquad \dots(8)$$

Ordering cost=A.

The present value of total holding cost during the inventory cycle is given by

$$C_H = c_1\left[\int_0^{T_0} q e^{-Rt} dt\right]$$

$$= c_1\left[\int_0^{\mu} q e^{-Rt} dt + \int_{\mu}^{T_0} q e^{-Rt} dt\right]$$

$$= c_1\left[\left(S+\frac{\alpha}{\beta}\right)\frac{(1-e^{-(\beta+R)\mu})}{(\beta+R)}+\frac{\alpha}{\beta R}(e^{-R\mu}-1)\right.$$
$$\left.+\frac{\alpha}{\theta+\gamma}\left(\frac{e^{(\theta+\gamma)(T_0-\mu)-R\mu}-e^{-RT_0}}{\theta+\gamma+R}\right)+\frac{1}{R}(e^{-RT_0}-e^{-R\mu})\right]. \qquad \ldots(9)$$

The present value of total shortage cost for the entire cycle is

$$C_S = c_2\left[\int_{T_0}^{T} q e^{-Rt}\,dt\right]$$
$$= c_2\left[\int_{T_0}^{T}\left\{-\alpha(t-T_0)+\frac{\delta}{2}(t^2-T_0^2)\right\}e^{-Rt}\,dt\right]$$
$$= c_2\left[(T-T_0)\left(\alpha-\frac{\delta}{2}(T+T_0)\right)\frac{e^{-RT}}{R}+(\alpha-\delta T)\frac{e^{-RT}}{R^2}-\frac{\delta e^{-RT}}{R^3}\right.$$
$$\left.-(\alpha-\delta T_0)\frac{e^{-RT_0}}{R^2}+\frac{\delta e^{-RT_0}}{R^3}\right]. \qquad \ldots(10)$$

Total deteriorated units per cycle are given by

$$D = S-\int_{\mu}^{T_0}(\alpha+\gamma q)\,dt$$
$$= S-\int_{\mu}^{T_0}\left(\alpha+\frac{\alpha\gamma}{\theta+\gamma}\left[e^{(\theta+\gamma)(T_0-t)}-1\right]\right)dt$$
$$= S-\frac{\alpha\theta}{(\theta+\gamma)}(T_0-\mu)+\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-\mu)}-1\right). \qquad \ldots(11)$$

The present value of deterioration cost is given by

$$C_D = ce^{-R\mu}\left(S-\frac{\alpha\theta}{(\theta+\gamma)}(T_0-\mu)+\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-\mu)}-1\right)\right). \qquad \ldots(12)$$

The total amount backordered at the end of each cycle is given by

$$Q-S = \alpha(t-T_0)-\frac{\delta}{2}(t^2-T_0^2). \qquad \ldots(13)$$

Now we consider the following two cases:

Case 1: $M \le T_0$.
Sub case I: $0 < M \le \mu$.

In this case, the present value of interest payable is given by

$$I_{p1}^1 = cI_c\left[\int_M^{\mu} q(t)e^{-Rt}\,dt+\int_{\mu}^{T_0} q(t)\,e^{-Rt}\,dt\right]$$
$$= cI_c\left[\int_M^{\mu}\left(\left(S+\frac{\alpha}{\beta}\right)e^{-\beta t}-\frac{\alpha}{\beta}\right)e^{-Rt}\,dt+\right.$$
$$\left.\int_{\mu}^{T_0}\frac{\alpha}{\theta+\gamma}\left(e^{(\theta+\gamma)(T_0-t)}-1\right)e^{-Rt}\,dt\right]$$
$$= cI_c\left[\frac{1}{\beta+R}\left(S+\frac{\alpha}{\beta}\right)\left(e^{-(\beta+R)M}-e^{-(\beta+R)\mu}\right)+\frac{\alpha}{\beta R}\left(e^{-R\mu}-e^{-RM}\right)\right.$$
$$\left.+\frac{\alpha}{\theta+\gamma}\left\{\frac{e^{(\theta+\gamma)(T_0-\mu)-R\mu}-e^{-RT_0}}{\theta+\gamma+R}+\frac{1}{R}\left(e^{-RT_0}-e^{-R\mu}\right)\right\}\right]. \qquad \ldots(14)$$

In this case, the present value of interest earned is given by

$$I_{e1}^1 = sI_e\left[\int_0^{\mu}(\alpha+\beta q)te^{-Rt}\,dt+\int_{\mu}^{T_0}(\alpha+\gamma q)te^{-Rt}\,dt\right]$$
$$= sI_e\left[\int_0^{\mu}\left(\alpha+\beta\left(\left(S+\frac{\alpha}{\beta}\right)e^{-\beta t}-\frac{\alpha}{\beta}\right)\right)te^{-Rt}\,dt+\int_{\mu}^{T_0}\left(\alpha+\gamma\frac{\alpha}{\theta+\gamma}\left[e^{(\theta+\gamma)(T_0-t)}-1\right]\right)te^{-Rt}\,dt\right]$$

$$= sI_e\left[\left(S+\frac{\alpha}{\beta}\right)\left\{-\frac{\mu e^{-(\beta+R)}}{(\beta+R)}+\frac{(1-e^{-(\beta+R)})}{(\beta+R)^2}\right\}\right.$$
$$+\frac{\alpha\theta}{\theta+\gamma}\left\{\left(\frac{\mu e^{-R\mu}-T_0e^{-RT_0}}{R}\right)+\left(\frac{e^{-R\mu}-e^{-RT_0}}{R^2}\right)\right\}$$
$$\left.+\frac{\alpha\gamma}{\theta+\gamma}\left\{\frac{\mu e^{(\theta+\gamma)(T_0-\mu)-R\mu}-T_0e^{-RT_0}}{\theta+\gamma+R}+\frac{e^{(\theta+\gamma)(T_0-\mu)-R\mu}-e^{-RT_0}}{(\theta+\gamma+R)^2}\right\}\right] \quad \ldots(15)$$

Therefore the present value of total cost during the entire cycle is given by

$$TC(1)=\left[A+C_H+C_S+C_D+I_{p1}^1-I_{e1}^1\right]$$
$$=A+c_1\left[\left(S+\frac{\alpha}{\beta}\right)\frac{(1-e^{-(\beta+R)\mu})}{(\beta+R)}+\frac{\alpha}{\beta R}(e^{-R\mu}-1)\right.$$
$$\left.+\frac{\alpha}{\theta+\gamma}\left(\frac{e^{(\theta+\gamma)(T_0-\mu)-R\mu}-e^{-RT_0}}{\theta+\gamma+R}\right)+\frac{1}{R}(e^{-RT_0}-e^{-R\mu})\right]$$
$$+c_2\left[(T-T_0)\left(\alpha-\frac{\delta}{2}(T+T_0)\right)\frac{e^{-RT}}{R}+(\alpha-\delta T)\frac{e^{-RT}}{R^2}-\frac{\delta e^{-RT}}{R^3}\right.$$
$$\left.-(\alpha-\delta T_0)\frac{e^{-RT_0}}{R^2}+\frac{\delta e^{-RT_0}}{R^3}\right]$$
$$+ce^{-R\mu}\left(S-\frac{\alpha\theta}{(\theta+\gamma)}(T_0-\mu)+\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-\mu)}-1\right)\right)$$
$$+cI_c\left[\frac{1}{\beta+R}\left(S+\frac{\alpha}{\beta}\right)\left(e^{-(\beta+R)M}-e^{-(\beta+R)\mu}\right)+\frac{\alpha}{\beta R}(e^{-R\mu}-e^{-RM})\right.$$
$$\left.+\frac{\alpha}{\theta+\gamma}\left\{\frac{e^{(\theta+\gamma)(T_0-\mu)-R\mu}-e^{-RT_0}}{\theta+\gamma+R}+\frac{1}{R}(e^{-RT_0}-e^{-R\mu})\right\}\right]$$
$$-sI_e\left[\left(S+\frac{\alpha}{\beta}\right)\left\{-\frac{\mu e^{-(\beta+R)}}{(\beta+R)}+\frac{(1-e^{-(\beta+R)})}{(\beta+R)^2}\right\}\right.$$
$$+\frac{\alpha\theta}{\theta+\gamma}\left\{\left(\frac{\mu e^{-R\mu}-T_0e^{-RT_0}}{R}\right)+\left(\frac{e^{-R\mu}-e^{-RT_0}}{R^2}\right)\right\}$$
$$\left.+\frac{\alpha\gamma}{\theta+\gamma}\left\{\frac{\mu e^{(\theta+\gamma)(T_0-\mu)-R\mu}-T_0e^{-RT_0}}{\theta+\gamma+R}+\frac{e^{(\theta+\gamma)(T_0-\mu)-R\mu}-e^{-RT_0}}{(\theta+\gamma+R)^2}\right\}\right]. \quad \ldots(16)$$

Sub case II: $\mu < M \le T_0$.

In this case, the present value of interest payable is given by

$$I_{p1}^2=cI_c\left[\int_M^{T_0}q(t)e^{-Rt}dt\right]$$
$$=cI_c\left[\int_M^{T_0}\frac{\alpha}{\theta+\gamma}\left(e^{(\theta+\gamma)(T_0-t)}-1\right)e^{-Rt}dt\right]$$
$$=\frac{cI_c\alpha}{\theta+\gamma}\left\{\frac{e^{(\theta+\gamma)(T_0-M)-RM}-e^{-RT_0}}{\theta+\gamma+R}+\frac{1}{R}(e^{-RT_0}-e^{-RM})\right\}. \quad \ldots(17)$$

In this case, the present value of interest earned is given by

$$I_{e1}^2=sI_e\left[\int_0^{\mu}(\alpha+\beta q)te^{-Rt}dt+\int_{\mu}^{T_0}(\alpha+\gamma q)te^{-Rt}dt\right]$$
$$=sI_e\left[\left(S+\frac{\alpha}{\beta}\right)\left\{-\frac{\mu e^{-(\beta+R)}}{(\beta+R)}+\frac{(1-e^{-(\beta+R)})}{(\beta+R)^2}\right\}\right.$$
$$+\frac{\alpha\theta}{\theta+\gamma}\left\{\left(\frac{\mu e^{-R\mu}-T_0e^{-RT_0}}{R}\right)+\left(\frac{e^{-R\mu}-e^{-RT_0}}{R^2}\right)\right\}$$
$$\left.+\frac{\alpha\gamma}{\theta+\gamma}\left\{\frac{\mu e^{(\theta+\gamma)(T_0-\mu)-R\mu}-T_0e^{-RT_0}}{\theta+\gamma+R}+\frac{e^{(\theta+\gamma)(T_0-\mu)-R\mu}-e^{-RT_0}}{(\theta+\gamma+R)^2}\right\}\right]. \quad \ldots(18)$$

The present value of total cost during the cycle time is given by

$$TC(2) = \left[A + C_H + C_S + C_D + I_{p1}^2 - I_{e1}^2\right]$$
$$= A + c_1\left[\left(S + \frac{\alpha}{\beta}\right)\frac{(1-e^{-(\beta+R)\mu})}{(\beta+R)} + \frac{\alpha}{\beta R}(e^{-R\mu} - 1)\right.$$
$$\left. + \frac{\alpha}{\theta+\gamma}\left(\frac{e^{(\theta+\gamma)(T_0-\mu)-R\mu} - e^{-RT_0}}{\theta+\gamma+R}\right) + \frac{1}{R}(e^{-RT_0} - e^{-R\mu})\right]$$
$$+c_2\left[(T - T_0)\left(\alpha - \frac{\delta}{2}(T + T_0)\right)\frac{e^{-RT}}{R} + (\alpha - \delta T)\frac{e^{-RT}}{R^2} - \frac{\delta e^{-RT}}{R^3}\right.$$
$$\left. -(\alpha - \delta T_0)\frac{e^{-RT_0}}{R^2} + \frac{\delta e^{-RT_0}}{R^3}\right]$$
$$+ce^{-R\mu}\left(S - \frac{\alpha\theta}{(\theta+\gamma)}(T_0 - \mu) + \frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-\mu)} - 1\right)\right)$$
$$+\frac{cI_c\alpha}{\theta+\gamma}\left\{\frac{e^{(\theta+\gamma)(T_0-M)-RM} - e^{-RT_0}}{\theta+\gamma+R} + \frac{1}{R}(e^{-RT_0} - e^{-RM})\right\}$$
$$-sI_e\left[\left(S + \frac{\alpha}{\beta}\right)\left\{-\frac{\mu e^{-(\beta+R)}}{(\beta+R)} + \frac{(1-e^{-(\beta+R)})}{(\beta+R)^2}\right\}\right.$$
$$+\frac{\alpha\theta}{\theta+\gamma}\left\{\left(\frac{\mu e^{-R\mu} - T_0e^{-RT_0}}{R}\right) + \left(\frac{e^{-R\mu} - e^{-RT_0}}{R^2}\right)\right\}$$
$$\left. +\frac{\alpha\gamma}{\theta+\gamma}\left\{\frac{\mu e^{(\theta+\gamma)(T_0-\mu)-R\mu} - T_0e^{-RT_0}}{\theta+\gamma+R} + \frac{e^{(\theta+\gamma)(T_0-\mu)-R\mu} - e^{-RT_0}}{(\theta+\gamma+R)^2}\right\}\right]. \quad \ldots(19)$$

Case 2: $M \geq T_0$.
In this case, there will be no interest charged.
The present value of interest earned is given by

$$I_{e2}^1 = sI_e\left[\int_0^\mu(\alpha + \beta q)te^{-Rt}dt + \int_\mu^{T_0}(\alpha + \gamma q)te^{-Rt}dt + (M - T_0)e^{-RT_0}\int_{T_0}^M(\alpha + \delta(t - T_0))tdt\right]$$
$$= sI_e\left[\left(S + \frac{\alpha}{\beta}\right)\left\{-\frac{\mu e^{-(\beta+R)}}{(\beta+R)} + \frac{(1-e^{-(\beta+R)})}{(\beta+R)^2}\right\}\right.$$
$$+\frac{\alpha\theta}{\theta+\gamma}\left\{\left(\frac{\mu e^{-R\mu} - T_0e^{-RT_0}}{R}\right) + \left(\frac{e^{-R\mu} - e^{-RT_0}}{R^2}\right)\right\}$$
$$+\frac{\alpha\gamma}{\theta+\gamma}\left\{\frac{\mu e^{(\theta+\gamma)(T_0-\mu)-R\mu} - T_0e^{-RT_0}}{\theta+\gamma+R} + \frac{e^{(\theta+\gamma)(T_0-\mu)-R\mu} - e^{-RT_0}}{(\theta+\gamma+R)^2}\right\}$$
$$\left. +(M - T_0)e^{-RT_0}\left[\frac{1}{2}(\alpha - \delta T_0)(M^2 - {T_0}^2) + \frac{\delta}{3}(M^3 - {T_0}^3)\right]\right]. \quad \ldots(20)$$

The present value of total cost during the cycle time is given by

$$TC(3) = [A + C_H + C_S + C_D - I_{e2}^1]$$
$$= A + c_1\left[\left(S + \frac{\alpha}{\beta}\right)\frac{(1-e^{-(\beta+R)\mu})}{(\beta+R)} + \frac{\alpha}{\beta R}(e^{-R\mu} - 1)\right.$$
$$\left. + \frac{\alpha}{\theta+\gamma}\left(\frac{e^{(\theta+\gamma)(T_0-\mu)-R\mu} - e^{-RT_0}}{\theta+\gamma+R}\right) + \frac{1}{R}(e^{-RT_0} - e^{-R\mu})\right]$$
$$+c_2\left[(T - T_0)\left(\alpha - \frac{\delta}{2}(T + T_0)\right)\frac{e^{-RT}}{R} + (\alpha - \delta T)\frac{e^{-RT}}{R^2} - \frac{\delta e^{-RT}}{R^3}\right.$$
$$\left. -(\alpha - \delta T_0)\frac{e^{-RT_0}}{R^2} + \frac{\delta e^{-RT_0}}{R^3}\right]$$
$$+ce^{-R\mu}\left(S - \frac{\alpha\theta}{(\theta+\gamma)}(T_0 - \mu) + \frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-\mu)} - 1\right)\right)$$

$$-sI_e\Bigg[\left(S+\frac{\alpha}{\beta}\right)\left\{-\frac{\mu e^{-(\beta+R)}}{(\beta+R)}+\frac{(1-e^{-(\beta+R)})}{(\beta+R)^2}\right\}$$
$$+\frac{\alpha\theta}{\theta+\gamma}\left\{\left(\frac{\mu e^{-R\mu}-T_0e^{-RT_0}}{R}\right)+\left(\frac{e^{-R\mu}-e^{-RT_0}}{R^2}\right)\right\}$$
$$+\frac{\alpha\gamma}{\theta+\gamma}\left\{\frac{\mu e^{(\theta+\gamma)(T_0-\mu)-R\mu}-T_0e^{-RT_0}}{\theta+\gamma+R}+\frac{e^{(\theta+\gamma)(T_0-\mu)-R\mu}-e^{-RT_0}}{(\theta+\gamma+R)^2}\right\}$$
$$+(M-T_0)e^{-RT_0}\left[\frac{1}{2}(\alpha-\delta T_0)\left(M^2-{T_0}^2\right)+\frac{\delta}{3}\left(M^3-{T_0}^3\right)\right]\Bigg]. \qquad \ldots(21)$$

Thus we have

$$TC=\begin{cases}TC(1), & 0<M\le\mu\\ TC(2), & \mu<M\le T_0\\ TC(3), & M\ge T_0\end{cases}$$

For optimization, we have

$$\frac{\partial TC}{\partial T_0}=0,\ \frac{\partial TC}{\partial T}=0,\ \left(\frac{\partial^2 TC}{\partial {T_0}^2}\right)\ge 0 \text{ and } \left(\frac{\partial^2 TC}{\partial {T_0}^2}\right).\left(\frac{\partial^2 TC}{\partial T^2}\right)-\left(\frac{\partial^2 TC}{\partial T_0\partial T}\right)^2\ge 0.$$

Now $\frac{\partial TC(1)}{\partial T_0}=0$ implies that

$$\frac{\alpha}{\theta+\gamma}\left(\frac{(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)-R\mu}-Re^{-RT_0}}{\theta+\gamma+R}\right)-e^{-RT_0}$$
$$+c_2\left[(-\alpha+\delta T_0)\frac{e^{-RT}}{R}+(\alpha-\delta T_0)\frac{e^{-RT_0}}{R}\right]$$
$$+ce^{-R\mu}\left(-\frac{\alpha\theta}{(\theta+\gamma)}+\frac{\alpha\gamma}{(\theta+\gamma)}e^{(\theta+\gamma)(T_0-\mu)}\right)$$
$$+cI_c\left[\frac{\alpha}{\theta+\gamma}\left(\frac{(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)-R\mu}-Re^{-RT_0}}{\theta+\gamma+R}\right)-e^{-RT_0}\right]$$
$$-sI_e\left[\frac{\alpha\theta}{\theta+\gamma}\left(e^{-RT_0}+\frac{e^{-RT_0}}{R}\right)+\frac{\alpha\gamma}{\theta+\gamma}\left\{\frac{\mu(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)-R\mu}+T_0Re^{-RT_0}}{\theta+\gamma+R}\right.\right.$$
$$\left.\left.+\frac{(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)-R\mu}}{(\theta+\gamma+R)^2}\right\}\right]=0, \qquad \ldots(22)$$

and $\frac{\partial TC(1)}{\partial T}=0$ implies that

$$c_2(T-T_0)\left(\alpha-\frac{\delta}{2}(T+T_0)\right)e^{-RT}=0\,. \qquad \ldots(23)$$

Similarly $\frac{\partial TC(2)}{\partial T_0}=0$ implies that

$$\frac{\alpha}{\theta+\gamma}\left(\frac{(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)-R\mu}-Re^{-RT_0}}{\theta+\gamma+R}\right)-e^{-RT_0}$$
$$+c_2\left[(-\alpha+\delta T_0)\frac{e^{-RT}}{R}+(\alpha-\delta T_0)\frac{e^{-RT_0}}{R}\right]$$
$$+ce^{-R\mu}\left(-\frac{\alpha\theta}{(\theta+\gamma)}+\frac{\alpha\gamma}{(\theta+\gamma)}e^{(\theta+\gamma)(T_0-\mu)}\right)$$
$$+cI_c\left[\frac{\alpha}{\theta+\gamma}\left(\frac{(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)-R\mu}-Re^{-RT_0}}{\theta+\gamma+R}\right)-e^{-RT_0}\right]$$
$$-sI_e\left[\frac{\alpha\theta}{\theta+\gamma}\left(e^{-RT_0}+\frac{e^{-RT_0}}{R}\right)+\frac{\alpha\gamma}{\theta+\gamma}\left\{\frac{\mu(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)-R\mu}+T_0Re^{-RT_0}}{\theta+\gamma+R}\right.\right.$$
$$\left.\left.+\frac{(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)-R\mu}}{(\theta+\gamma+R)^2}\right\}\right]=0, \qquad \ldots(24\ \ )$$

and $\frac{\partial TC(2)}{\partial T}=0$ implies that

$$c_2(T-T_0)\left(\alpha-\frac{\delta}{2}(T+T_0)\right)e^{-RT}=0\,. \qquad \ldots(25)$$

Similarly $\frac{\partial TC(3)}{\partial T_0}=0$ implies that

$$\frac{\alpha}{\theta+\gamma}\left(\frac{(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)-R\mu}-Re^{-RT_0}}{\theta+\gamma+R}\right)-e^{-RT_0}$$

$$+c_2\left[(-\alpha+\delta T_0)\frac{e^{-RT}}{R}+(\alpha-\delta T_0)\frac{e^{-RT_0}}{R}\right]\ +ce^{-R\mu}\left(-\frac{\alpha\theta}{(\theta+\gamma)}+\frac{\alpha\gamma}{(\theta+\gamma)}e^{(\theta+\gamma)(T_0-\mu)}\right)$$

$$+cI_c\left[\frac{\alpha}{\theta+\gamma}\left(\frac{(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)-R\mu}-Re^{-RT_0}}{\theta+\gamma+R}\right)-e^{-RT_0}\right]$$

$$-sI_e\left[\frac{\alpha\theta}{\theta+\gamma}\left(e^{-RT_0}+\frac{e^{-RT_0}}{R}\right)+\frac{\alpha\gamma}{\theta+\gamma}\left\{\frac{\mu(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)-R\mu}+T_0Re^{-RT_0}}{\theta+\gamma+R}+\frac{(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)-R\mu}}{(\theta+\gamma+R)^2}\right\}\right.$$

$$-(R(M-T_0)+1)e^{-RT_0}\left[\frac{1}{2}(\alpha-\delta T_0)(M^2-T_0{}^2)+\frac{\delta}{3}(M^3-T_0{}^3)\right]$$

$$\left.-(M-T_0)e^{-RT_0}\left[T_0(\alpha-\delta T_0)+\delta T_0{}^2-\frac{\delta}{2}(M^2-T_0{}^2)\right]\right]=0, \qquad \ldots(26)$$

and $\frac{\partial TC(3)}{\partial T}=0$ implies that

$$c_2(T-T_0)\left(\alpha-\frac{\delta}{2}(T+T_0)\right)e^{-RT}=0. \qquad \ldots(27)$$

Solving these equations we can obtained the values of T and T_0. It can be shown that these values satisfy $\left(\frac{\partial^2 TC}{\partial T_0{}^2}\right)\geq 0$ and $\left(\frac{\partial^2 TC}{\partial T_0{}^2}\right).\left(\frac{\partial^2 TC}{\partial T^2}\right)-\left(\frac{\partial^2 TC}{\partial T_0\,\partial T}\right)^2\geq 0$. Thus the values of T and T_0 obtained above are the optimal values. These may be denoted by T* and T_0^*. Substituting these values in the expression (8) and (13), we can obtain the optimal values of Q* and S*. Mathematika or matlab software can be used for illustration of the model.

CONCLUDING REMARKS:

In this paper, an EOQ model has been developed for deteriorating products with life time under trade credit. Three components demand rate has been considered. During life time, the demand rate is stock dependent. During deterioration period, the demand rate is again stock dependent but with decreased rate. During shortage period the demand rate is time dependent. Deterioration rate has been taken constant. Permissible delay in payments is allowed and inflation has been considered. Permissible delay in payments has been considered in three cases. This model can further be developed for other forms of demand rate and variable rate of deterioration.

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